Résolution du problème

```
> restart;
```

Equation de la chaînette

```
> equation1:=mu*g/H*sqrt(1+diff(y(x),x)^2)=diff(y(x),x$2):
> ini1:=y(0)=0,D(y)(0)=0:
> solution1:=op(1,[dsolve({equation1,ini1})]):
> y(x):=op(2,solution1);C:=y(x):
```

$$y(x) := \frac{H\left(\cosh\left(\frac{\mu\, g\, x}{H}\right) - 1\right)}{\mu\, g}$$

```
> h:=subs(x=L/2,y(x));
```

$$h := \frac{H\left(\cosh\left(\frac{\mu\, g\, L}{2\, H}\right) - 1\right)}{\mu\, g}$$

Expression de la longueur de la chaîne

```
> equation2:=diff(l(x),x)=sqrt(1+diff(y(x),x)^2):
> ini2:=l(-L/2)=0:
> solution2:=dsolve({equation2,ini2},l(x)):
> longueur:=l=simplify(subs(x=L/2,op(2,solution2))):
> assume(Re(cosh(1/2*g*mu/H*L))>0);
> longueur;
```

$$l = \frac{2\, H\sim\, \sinh\left(\frac{\mu\sim\, g\sim\, L\sim}{2\, H\sim}\right)}{\mu\sim\, g\sim}$$

Procédure

```
> chainette:=proc(mu,g,l,L)
  local longueur,H,y;
  longueur := l = 2*H*sinh(1/2*mu*g/H*L)/mu/g;
  H:=abs(fsolve(longueur,H));
  y(x):=H*(cosh(mu*g/H*x)-1)/mu/g;
  end:
```

Tracé

```
> courbe_chainette:=proc(mu,g,l,L)
  local i,courbe,h;
  courbe:=chainette(0.01,9.8,1,L);
  h:=eval(subs(x=L/2,courbe(x)));
  plot(courbe,x=-L/2..L/2,y=0..h);
  end:
```

```
> courbe_chainette_parabole:=proc(mu,g,l,L)
  local i,courbe,h,a,b;
  courbe:=chainette(0.01,9.8,1,L);
  h:=eval(subs(x=L/2,courbe(x)));
  b:=solve(a*(L/2)^2=h,a);
  plot([courbe,b*(x)**2],x=-L/2..L/2,y=0..h,scaling=constrained);
  end;
```

courbe_chainette_parabole := **proc**(μ, *g*, *l*, *L*)

local *i*, *courbe*, *h*, *a*, *b*;

courbe := chainette(0.01, 9.8, 1, *L*);

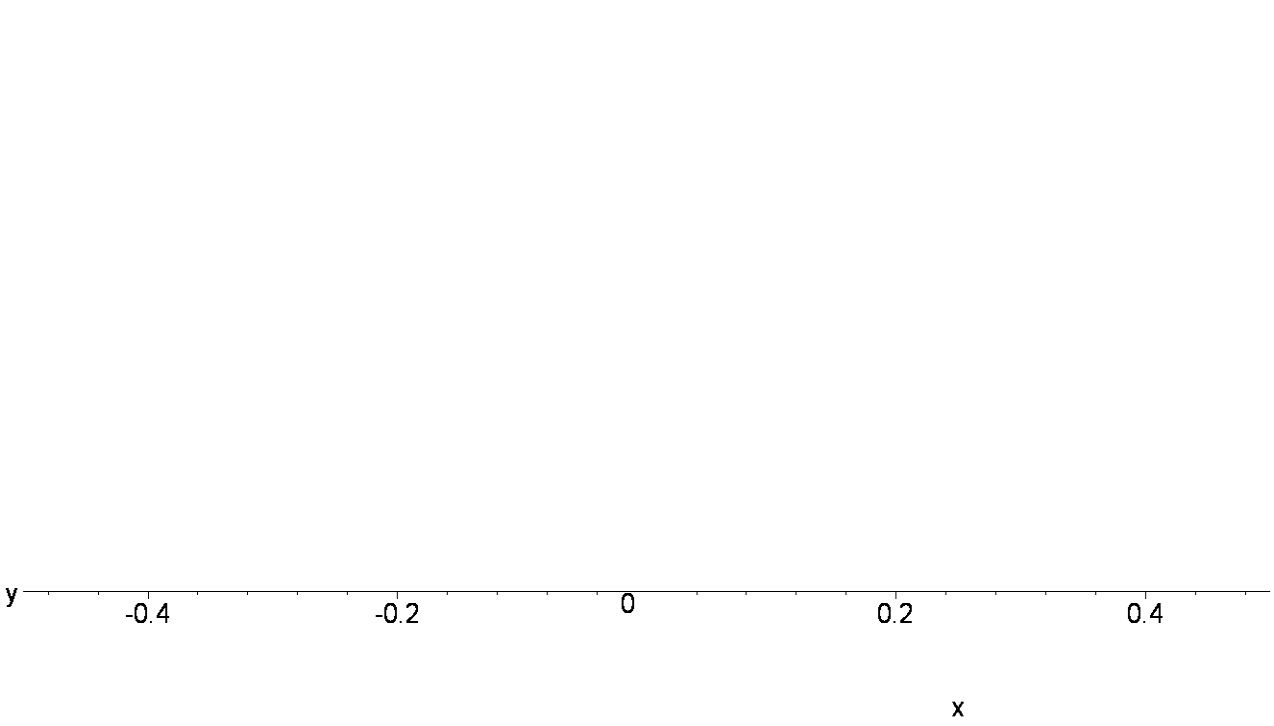
h := eval(subs(*x* = 1 / 2∗*L*, *courbe*(*x*)));

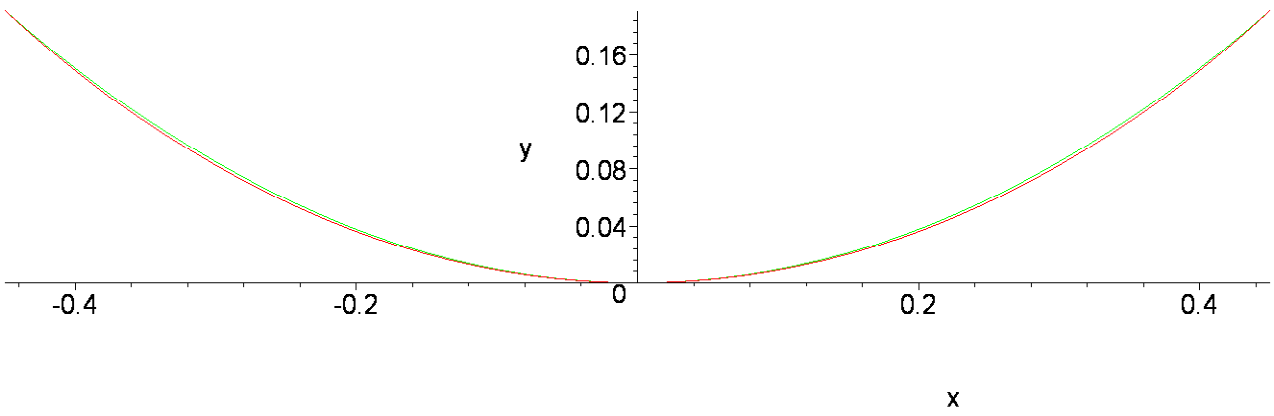
b := solve(1 / 4∗*a*∗*L*^2 = *h*, *a*);

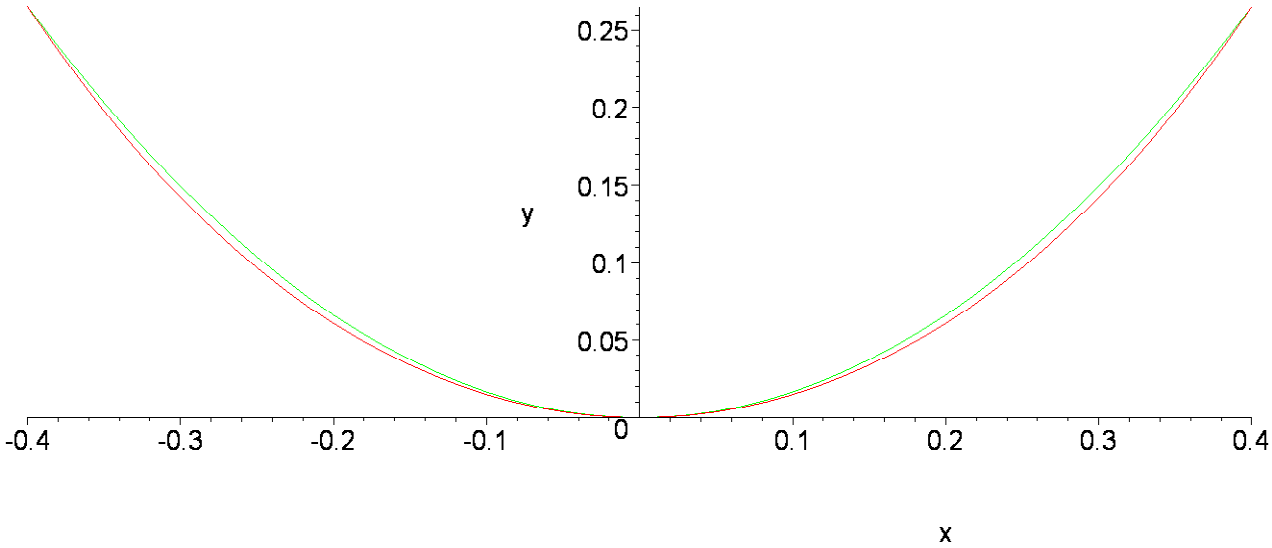
plot([*courbe*, *b*∗*x*^2], *x* = −1 / 2∗*L* .. 1 / 2∗*L*, *y* = 0 .. *h*, *scaling* = *constrained*)

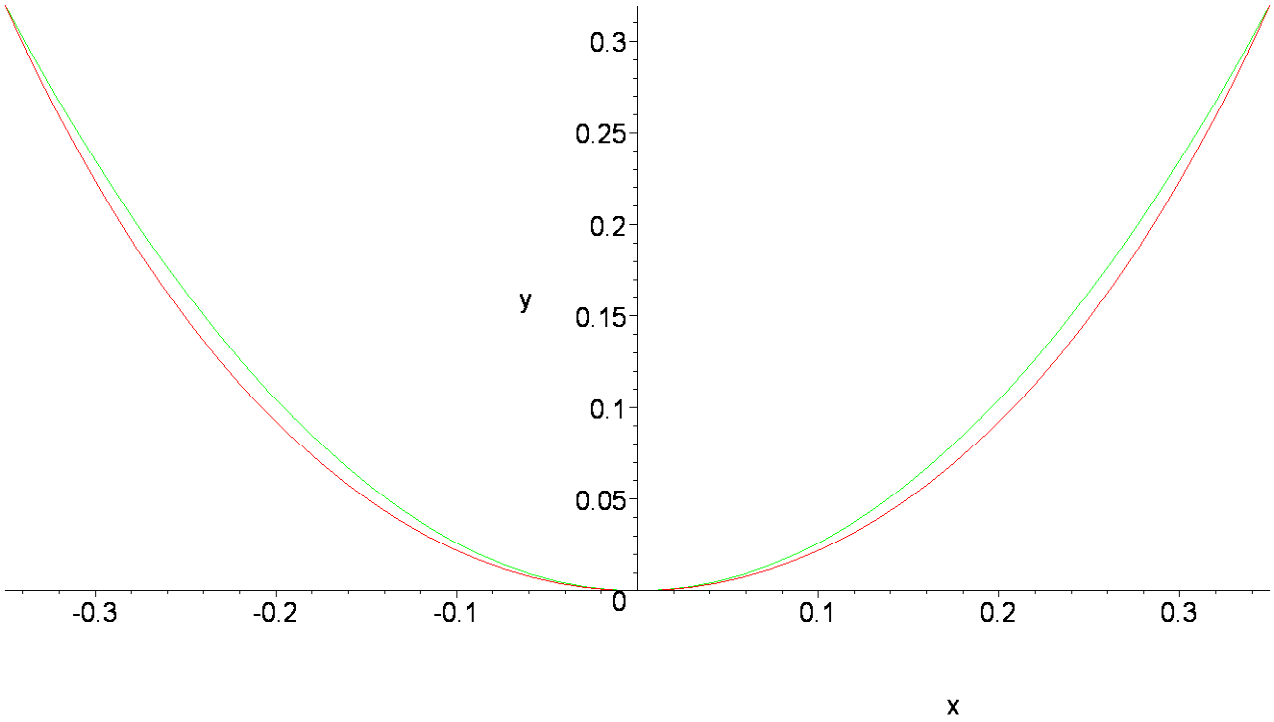
end proc

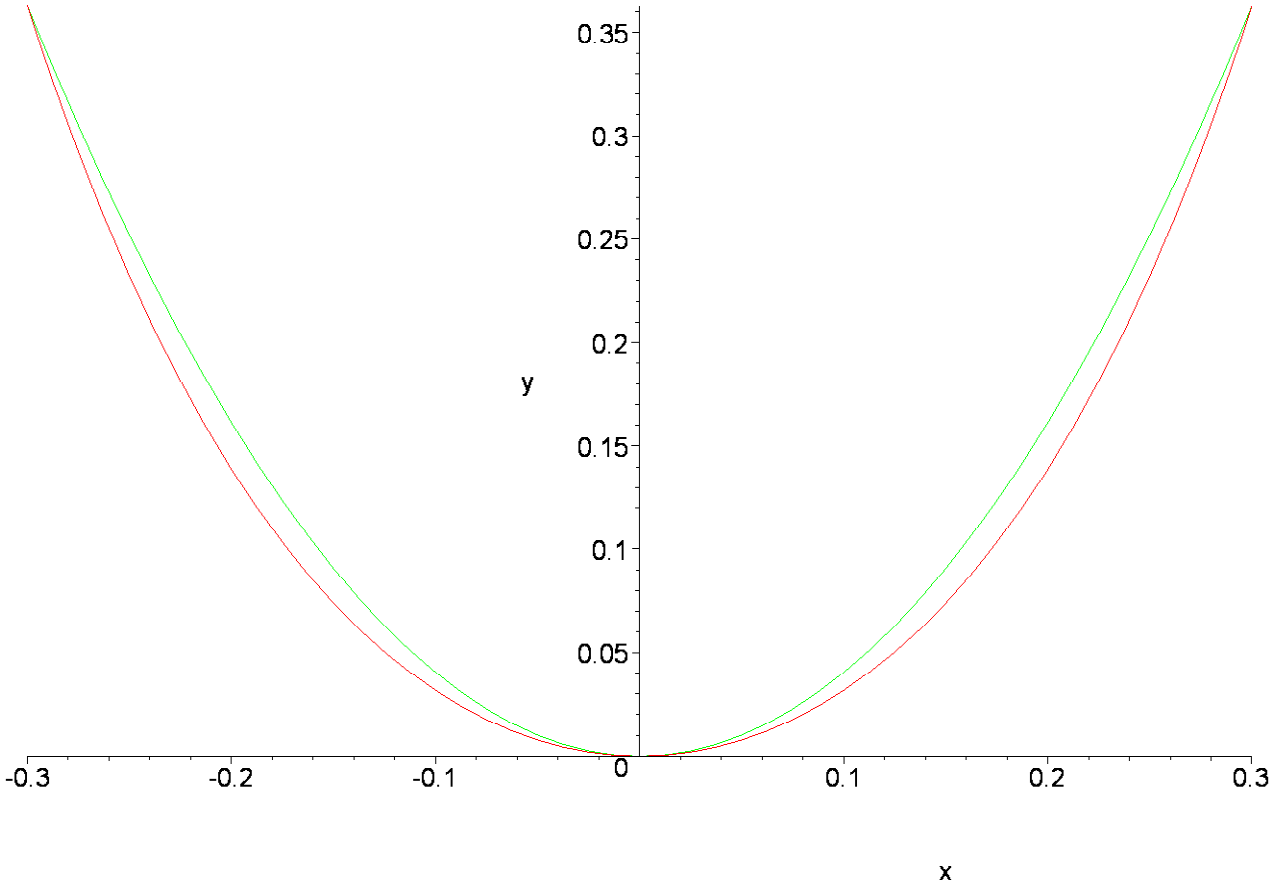
```
> LL:=[1.0,0.9,0.8,0.7,0.6,0.5,0.4]:
  for i from 1 to nops(LL) do
    courbe_chainette_parabole(0.01,9.8,1,LL[i]);
  od;
```

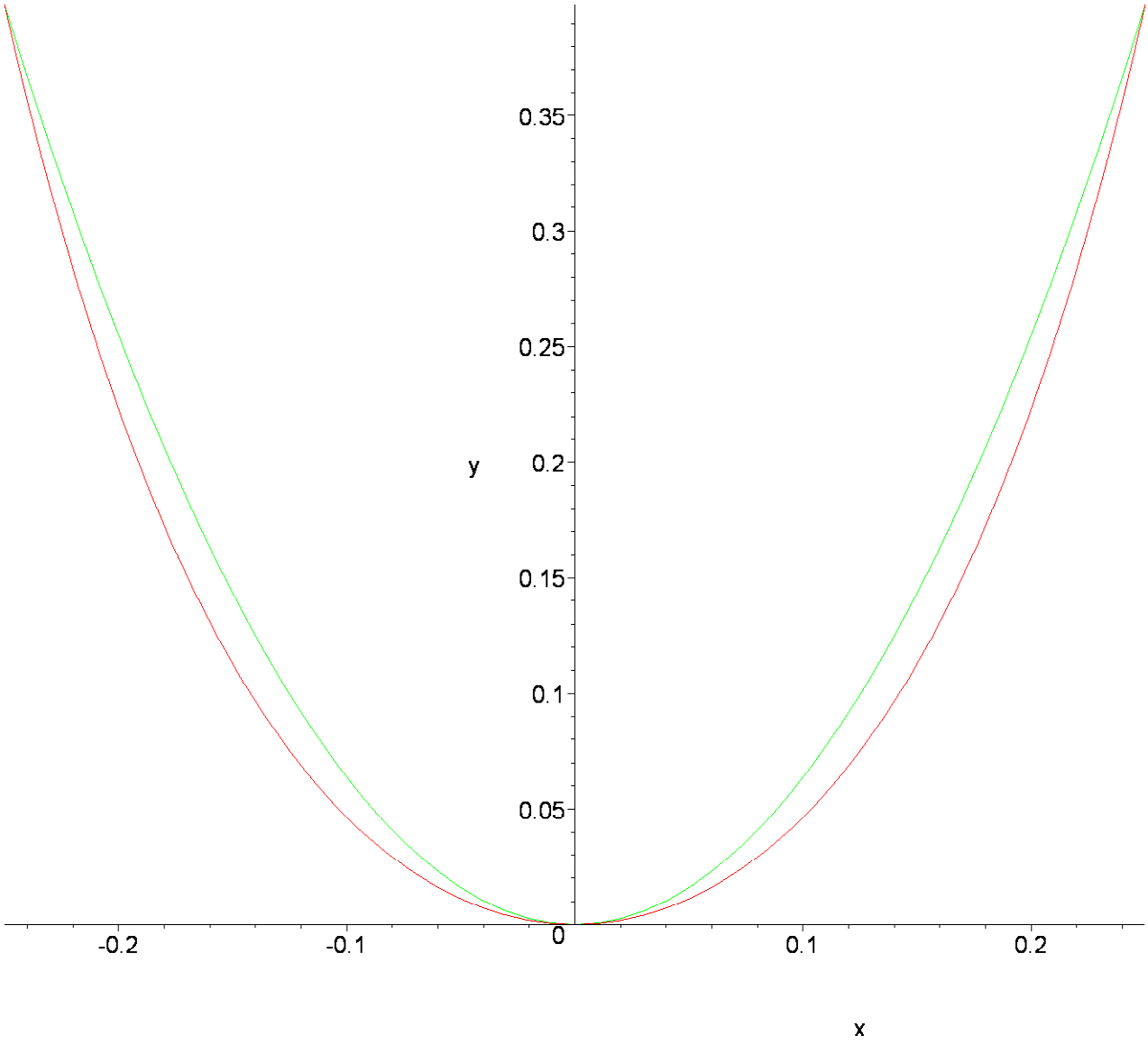

y
-0.4
-0.2
0
0.2
0.4
x

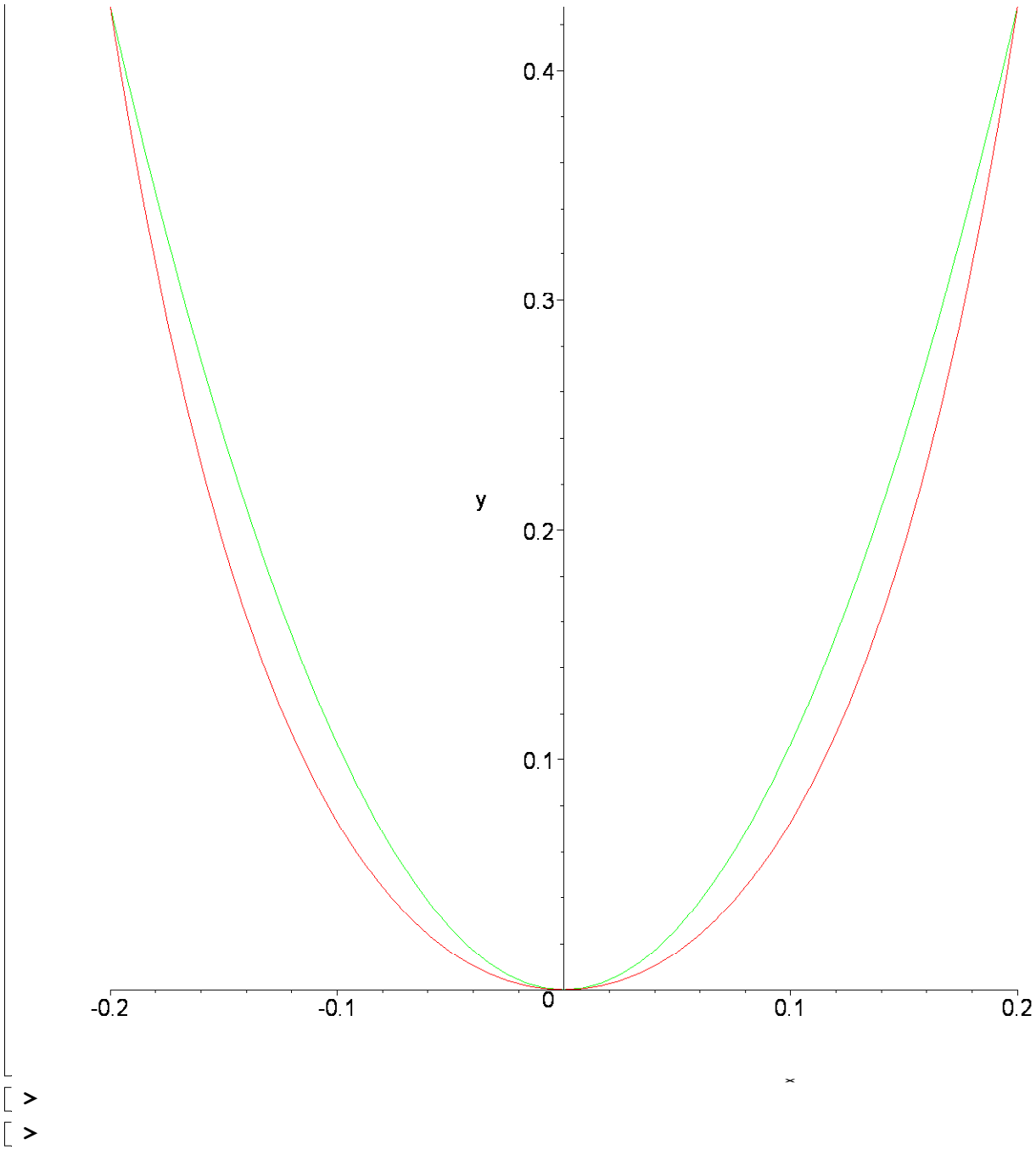

0.16
0.12
y
0.08
0.04
-0.4
-0.2
0
0.2
0.4
x


0.25
0.2
0.15
y
0.1
0.05
0
-0.4
-0.3
-0.2
-0.1
0.1
0.2
0.3
0.4
x


0.3
0.25
0.2
y
0.15
0.1
0.05
0
-0.3
-0.2
-0.1
0.1
0.2
0.3
x


0.35
0.3
0.25
0.2
y
0.15
0.1
0.05
0
-0.3
-0.2
-0.1
0.1
0.2
0.3
x


0.35
0.3
0.25
y
0.2
0.15
0.1
0.05
0
-0.2
-0.1
0.1
0.2
x


0.4
0.3
y
0.2
0.1
-0.2
-0.1
0
0.1
0.2
x

>

>